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Perfect light transmission in Fibonacci arrays of dielectric multilayers

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Abstract
In this paper we study the propagation of light through an asymmetric array of dielectric multilayers built by joining two porous silicon substructures in a Fibonacci sequence. Each Fibonacci substructure follows the well-known recursive rule but in the second substructure dielectric layers A and B are exchanged. Even without mirror symmetry, this array gives rise to multiple transparent states, which follow the scaling properties and self-similar spectra of a single Fibonacci multilayer. We apply the transfer matrix formalism to calculate the transmittance. By setting the transfer matrix of the array equal to $\pm I$, the identity matrix, frequencies of perfect light transmission are reproduced in our theoretical calculations. Although the light absorption of porous silicon in the optical range limits our experimental study to low Fibonacci generations, the positions of the transparent states are well predicted by the above-mentioned condition. We conclude that mirror symmetry in arrays of Fibonacci multilayers is sufficient but not necessary to generate multiple transparent states, opening broader applications of quasiperiodic systems as filters and microcavities of multiple frequencies.

1. Introduction

Quasicrystals are perfect non-periodically ordered materials with a long-range symmetry that show interesting electronic properties such as self-similar spectra and critical states. They do not present either extended Bloch-like states, as in periodic crystals, or localized states, as in random systems [1]. Their wavefunctions are not localized exponentially but only weakly localized and have a rich structure including scaling with various indexes, i.e. a multifractal nature [2]. To fully appreciate the specific features of quasiperiodic systems arising from their fractal nature, the study of classical waves in one dimension offers a number of advantages over the study of quantum elementary excitations since the presence of electron–phonon, electron–electron, or spin–orbit interaction analysis is difficult to include [3]. Within this approach, Kohmoto *et al* [3, 4] proposed dielectric multilayers in the Fibonacci sequence to study light localization and showed that the transmission coefficient has a multifractal spectrum due to its non-periodic long-range order. These results spurred interest both for optical applications [5, 6] and theoretical aspects of light transmission in quasiperiodic media [7–11] pointing towards the possible localization of light by the effect of the quasiperiodic order.

Following these ideas, Iguchi [12] and later Maciá [10] provided further evidence of the rich behavior of light propagating through quasiperiodic media. They found that when the light phase inside each layer is equal to $n\pi$, the transfer matrices of a Fibonacci dielectric multilayer commute and transparent states appear. This means that any sequence at the corresponding frequency shows transparent states as a result of local resonances. However, transparent states can be present at different light phase conditions. For example, Huang *et al* [13] proposed an interesting array of two Fibonacci dielectric multilayers with internal mirror symmetry. Multiple perfect transmission peaks are found, which preserve the self-similarity of the transmission spectra and scaling factor of single Fibonacci dielectric multilayers. They suggested that the transparent states are a consequence of the internal mirror symmetry of the structure [14]. From the theoretical point of view, perfect transmission peaks are a manifestation of extended optical states present in quasiperiodic systems. Within this context, He *et al* [15] calculated the transmission

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spectra of symmetrical Fibonacci superlattices composed of positive and negative refractive index materials and they consider that localized states exist because mirror symmetry structure favors wave interference. Recently, Mauriz *et al* [16] studied the transmission properties of light through the symmetric Fibonacci photonic multilayers, made up of both positive (SiO2) and negative refractive index materials with a mirror symmetry, in which many perfect transmission peaks are numerically obtained and the transmission coefficient exhibits a six-cycle self-similar behavior. Generally speaking, there is an increasing interest in the development of photonic bandgap materials that transmit light in a narrow wavelength without losses [17]. Up to date, the best structures to perform this task are those with highly symmetrical Brillouin zones [17]. Although strictly speaking quasiperiodic structures do not exhibit Brillouin zones, they have an effective high symmetry averaged Brillouin zone. This idea has been tested in a tridimensional Penrose tiling made of a polymer for electromagnetic radiation in the microwave range [18]. Also, the propagation dynamics of waves in an optical quasicrystal have been studied using lasers in a photosensitive material [19], and theoretical studies have shown that quasiperiodicity can be used to build high reflectance cavities [20]. Such experiments and ideas suggest that quasiperiodic structures have a great potential in optical applications.

Encouraged by these results, we wanted to understand the origin of transparent states in Fibonacci arrays and proposed a structure of Fibonacci dielectric multilayers built by joining two Fibonacci sequences without internal symmetry. Surprisingly we found multiple transparent states similar to those obtained in multilayers with mirror symmetry, but at different positions in the transmittance spectrum. Furthermore, the asymmetric arrays preserve self-similar spectra with the scaling factor predicted by Kohmoto *et al* for a single Fibonacci dielectric multilayer [21]. By applying the transfer matrix formalism to the asymmetric Fibonacci array, light phases with perfect transmission are found when the transfer matrix is equal to the identity matrix ($\pm I$). These results provide evidence that the mirror symmetry is sufficient but not necessary to generate multiple transparent states in Fibonacci dielectric arrays.

In this study, porous silicon (PS) multilayers were produced to see experimentally the light propagation through asymmetric Fibonacci arrays. PS is a nanostructured material prepared by the electrochemical etching of crystalline silicon in an HF solution [22]. This process gives rise to a sponge-like structure with a crystalline skeleton surrounded by air. The fact that the electrochemical attack is self-limiting and occurs mainly in correspondence with pore tips allows us to build blocks of different porosity by alternating the applied currents, therefore alternating different refractive indexes [23, 24].

In section 2, the proposed asymmetric array of dielectric layers following the Fibonacci sequence as well as the transfer matrix formalism used to study the light propagation through these structures are described. In section 3, the experimental procedure to produce an asymmetric Fibonacci multilayer made of PS is given. In section 4 the theoretical and experimental results are presented. Finally, in section 5 the conclusions of this work are summarized.

2. Theory

In order to describe light propagation through a dielectric structure following a Fibonacci sequence, we use the classical model of the transfer matrix [25] and the notation introduced by Kohmoto *et al* [3, 4]. Let us consider that the multilayer is composed of two types of dielectric materials A and B, with refractive index and thickness η_A, η_B, d_A, and d_B respectively. An asymmetric array can be built by joining two substructures following the Fibonacci sequence. Each substructure follows the well-known recursive rule $F_j = \{F_{j-1}, F_{j-2}\}$ for $j > 1$. That is, the first Fibonacci substructure (F_j) starts with $F_0 = \{\mathrm{B}\}$ and $F_1 = \{\mathrm{A}\}$, such that $F_2 = \{\mathrm{AB}\}$, $F_3 = \{\mathrm{ABA}\}$, $F_4 = \{\mathrm{ABAAB}\}$, and so on. In the second substructure the same recursive formula is applied but we exchange A $\Leftrightarrow$ B, which we call 'conjugated Fibonacci' (C_j). In this way, $C_j = \{C_{j-1}, C_{j-2}\}$, with $C_0 = \{\mathrm{A}\}$ and $C_1 = \{\mathrm{B}\}$, such that $C_2 = \{\mathrm{BA}\}$, $C_3 = \{\mathrm{BAB}\}$, $C_4 = \{\mathrm{BABBA}\}$, etc. Now, the asymmetric array is constructed by joining the two substructures as $S_j = \{F_j \mid C_j\}$, such that one has $S_0 = \{\mathrm{B} \mid \mathrm{B}\}$, $S_1 = \{\mathrm{A} \mid \mathrm{A}\}$, $S_2 = \{\mathrm{AB} \mid \mathrm{BA}\}$, $S_3 = \{\mathrm{ABA} \mid \mathrm{BAB}\}$, $S_4 = \{\mathrm{ABAAB} \mid \mathrm{BABBA}\}$, etc. The number of layers of the joined array is twice the Fibonacci number (2, 4, 6, 10, 16, ...). Notice that in general there is no internal mirror symmetry with respect to the joining interface, denoted symbolically by (|). It is worth mentioning that if the conjugate Fibonacci multilayer is constructed by inverting from the last layer to the first one, that is $C_j = \{C_{j-2}, C_{j-1}\}$, and joining it to F_j, then there is an internal mirror symmetry, similar to the structure already studied by Huang *et al* [13, 14].

Now, let us consider the propagation of light with a polarization perpendicular to the light path (TE wave). Within the transfer matrix formalism [3, 25], the light propagation across the interface between layer A and layer B is given by the matrices:

$$T_{\mathrm{A|B}} = \begin{pmatrix} 1 & 0 \\ 0 & u^{-1} \end{pmatrix} \tag{1}$$

and

$$T_{\mathrm{B|A}} = T_{\mathrm{A|B}}^{-1}, \tag{2}$$

where $u = \eta_A \cos\theta_A / \eta_B \cos\theta_B$, with θ_A and θ_B being the incidence angle at layers A and B respectively. At normal incidence, u is a measure of the refractive index contrast between the dielectrics. Now, the light propagation within each layer A (or B) is expressed as

$$T_{\mathrm{A(B)}} = \begin{pmatrix} \cos\delta_{\mathrm{A(B)}} & -\sin\delta_{\mathrm{A(B)}} \\ \sin\delta_{\mathrm{A(B)}} & \cos\delta_{\mathrm{A(B)}} \end{pmatrix}, \tag{3}$$

where $\delta_{\mathrm{A(B)}} = 2\pi d_{\mathrm{A(B)}} \eta_{\mathrm{A(B)}} / \lambda \cos\theta_{\mathrm{A(B)}}$ is the light phase difference in layers A (or B). In this way, the light propagation through the entire multilayer can be described by the transfer matrix (M), which is the product of the interface matrices ($T_{\mathrm{A|B}}$ and $T_{\mathrm{B|A}}$) and the internal matrices (T_A and T_B). From M, the transmittance (T) and reflectance (R) are calculated as [3]

$$T = \frac{4}{|M|^2 + 2\det(M)} \tag{4}$$

and

$$R = \frac{|M|^2 - 2\det(M)}{|M|^2 + 2\det(M)}, \tag{5}$$

where $|M|^2$ is the sum of the squares of the four elements of the transfer matrix M and $\det(M)$ is the determinant of M.

At this step it is convenient to use renormalized transfer matrices by defining Q_A and Q_B as

$$Q_A = T_A \tag{6}$$

and

$$Q_B = T_{A|B} T_B T_{B|A} = \begin{pmatrix} \cos\delta_B & -u\sin\delta_B \\ u^{-1}\sin\delta_B & \cos\delta_B \end{pmatrix}. \tag{7}$$

Notice that Q_A and Q_B are unimodal matrices, thus the determinant of their product is also unimodal, which simplifies the calculation of T and R. Without loss of generality, one can consider a finite Fibonacci multilayer inside a media A, such that the transfer matrices of the Fibonacci and Fibonacci conjugate generations are given by the recursive formula

$$Q_{F_j} = Q_{F_{j-1}} Q_{F_{j-2}} \tag{8}$$

and

$$Q_{C_j} = Q_{C_{j-1}} Q_{C_{j-2}}. \tag{9}$$

In this way, the transfer matrix of the asymmetric array is expressed as

$$M_{S_j} = Q_{F_j} Q_{C_j}, \tag{10}$$

with the initial conditions $Q_{F_0} = Q_{C_1} = Q_B$ and $Q_{F_1} = Q_{C_0} = Q_A$.

By using the fact that $\det(M_{S_j}) = 1$, the transmittance can be expressed in terms of the trace $x = (m_{11} + m_{22})$ and the antitrace $y = (m_{12} - m_{21})$ of the transfer matrix M as

$$T = \frac{4}{x^2 + y^2}. \tag{11}$$

From this last equation, it can be seen that one possible solution for a perfect transmission ($T = 1$) can occur when $x = \pm 2$ and $y = 0$. A matrix that satisfies this condition is the identity matrix $\pm I$. This means that at the position of some perfect transmission peaks, $Q_{F_j} = \pm Q_{C_j}^{-1}$. It is worth mentioning that in general $Q_{F_j} \neq \pm Q_{C_j}^{-1}$, only at some light phases δ is the transfer matrix M_{S_j} equal to the identity matrix.

3. Experiment

To study the light transmission of asymmetric Fibonacci multilayers in real systems, at least at low generations, we chose PS. This material was prepared by standard electrochemical dissolution [22, 23]. Highly boron-doped silicon substrates (c-Si) with orientation (100) and electrical resistivity of 0.001–0.005 Ω cm were used. On one side of the c-Si wafer was deposited an aluminum film and it was then heated at 550 °C for 15 min to make electrical contact. The other side of the substrate was electrochemically etched in an aqueous solution of HF, ethanol, and glycerol with a volume ratio of 3:7:1 inside a Teflon cell. In addition, in order to keep the electrolyte during the electrochemical dissolution homogeneous, a peristaltic pump was used to circulate the etching solution. The electrochemical attack of the c-Si substrate starts by applying a constant current between the electrolyte and the wafer. The multilayer is produced by alternating the applied currents in the Fibonacci sequence, i.e. $J_A = 1.7$ mA cm^{-2} for the high refractive index layers and $J_B = 45$ mA cm^{-2} for the low refractive index ones. At these density currents, the refractive indexes of PS are approximately $\eta_A = 2.12$ and $\eta_B = 1.45$. For each layer at normal incidence we must have the light phase $\delta_A = \delta_B = \delta$, for a quarter wave of the optical path length, i.e. $\eta_A d_A = \eta_B d_B = \lambda_0/4$ at a central wavelength $\lambda_0 = 600$ nm. This means the layer thickness must be $d_A = 70$ nm and $d_B = 103$ nm. Given a substrate type, an electrolyte composition and a current density, the layer thickness of PS is a function of time. In order to get as close as possible to the quarter wavelength condition, a computer controlled the electrochemical process. As it is well known, PS has a large surface area that makes it sensitive to chemical reaction with elements present in the atmosphere. For this reason, it was necessary to passivate the surface after the electrochemical attack by thermally oxidizing the samples at 300 °C for 15 min. As the multilayer structure is built on the top of the c-Si substrate, we measured reflectance instead of transmittance. The reflectance spectra of samples were measured at room temperature with a Shimadzu spectrophotometer at an angle of incidence of 5°.

4. Results

In this section we describe the theoretical and experimental results of the light propagation through our asymmetric Fibonacci multilayer. We use the transfer matrix formalism to calculate transmittance spectra and PS multilayers as experimental samples.

4.1. Theoretical details

The transmission of light through the arrays S_j is calculated from equation (11). In the numerical calculation a normal incidence and refractive indexes $\eta_A = 2.12$ and $\eta_B = 1.45$ were considered. The optical path length of each layer was chosen to fulfil a quarter wavelength condition $\eta_A d_A = \eta_B d_B = \lambda_0/4$ with λ_0 a central wavelength (i.e. $\delta_A = \delta_B = \delta$ at normal incidence). In figure 1 the transmittance is shown as a function of λ_0/λ for generations: (a) S_5, (b) S_6, and (c) S_7 (solid lines), corresponding to 16, 26, and 42, layers, respectively. As can be observed from figure 1, the asymmetric Fibonacci array S_j gives rise to multiple perfect transmission peaks that are absent in the single Fibonacci multilayers F_j and C_j (dotted lines). However, the spectra preserve the quasiperiodic features of single Fibonacci multilayers: the self-similar spectra every six generations, $M_{S_j} = M_{S_{j+6}}$ for any j and the scaling factor around $\lambda_0/\lambda = 1$ [21]. These features come from the trace properties of the transfer matrices for single Fibonacci multilayers given by equation (8) with the

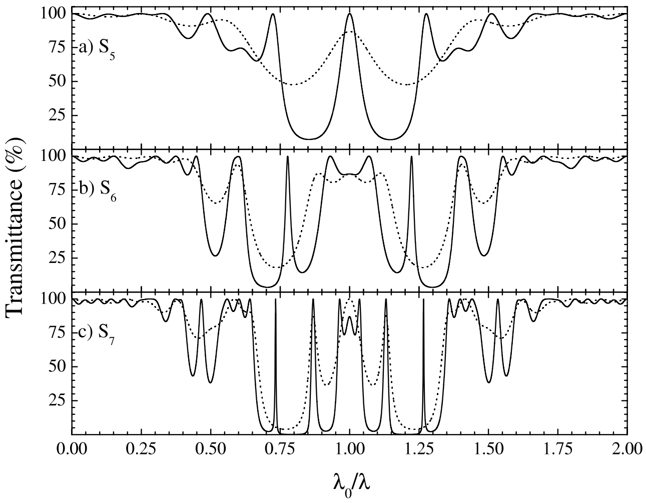

Figure 1. Comparison between the transmittance spectra for single Fibonacci (dotted line) and asymmetric Fibonacci (solid line) multilayers of generations (a) S_5, (b) S_6, and (c) S_7. In the numerical calculations normal incidence values $\eta_A = 2.12$ and $\eta_B = 1.45$ were considered. Notice that some transparent states in the asymmetric array are placed inside the gaps of the single Fibonacci multilayers.

initial conditions Q_A and Q_B, which can be considered as a dynamical map with a constant of motion (J) given by [3, 4]:

$$J = \frac{1}{4}\sin^2\delta_A \sin^2\delta_B\left(\frac{\eta_A\cos\theta_A}{\eta_B\cos\theta_B} - \frac{\eta_B\cos\theta_B}{\eta_A\cos\theta_A}\right)^2. \qquad (12)$$

This constant of motion is always positive and measures the strength of the quasiperiodicity. In particular, when $\delta_A = \delta_B = (m + \frac{1}{2})\pi$ with m an integer, i.e. $\lambda_0/\lambda = 2m + 1$, $J = (u^2 - 1)^2/4u^2$ and the quasiperiodicity is most effective [3]. At these light phases, the dynamical map has a six-cycle period, where it shows auto-similarity under a change of scale. The scaling factor of the transmission coefficient can be exactly calculated as [21],

$$K = \left(\sqrt{[1 + 4(1 + J)^2]} + 2(1 + J)\right)^2. \qquad (13)$$

The scale factor for the asymmetric Fibonacci array calculated from equation (13) by considering normal incidence and the above-mentioned refractive indexes gives $K = 23.16$. In figure 2 the self-similarities of the asymmetric array S_j (solid line) and of a single Fibonacci multilayer F_j (dotted line) for generations 8 and 14 are shown. Notice the difference of scale between figures (a) S_8 and (b) S_{14}.

These spectra are like those reported for mirror symmetric Fibonacci arrays. The mirror symmetric array is constructed by joining two Fibonacci substructures as $F_{M_j} = \{F_{j-1}, F_{j-2}, F_{j-2}, F_{j-1}\}$ [13, 14]. For the purpose of comparison, in figure 3 the transmittance spectra of the asymmetric array (a) S_j and the mirror symmetric array (b) F_{Mj} for generation 6 are shown. Both spectra are similar, but the positions of perfect transmission peaks are different. That is, perfect transmission peaks occur at different light phase conditions, even when one has considered $\delta_A = \delta_B$ in both arrays.

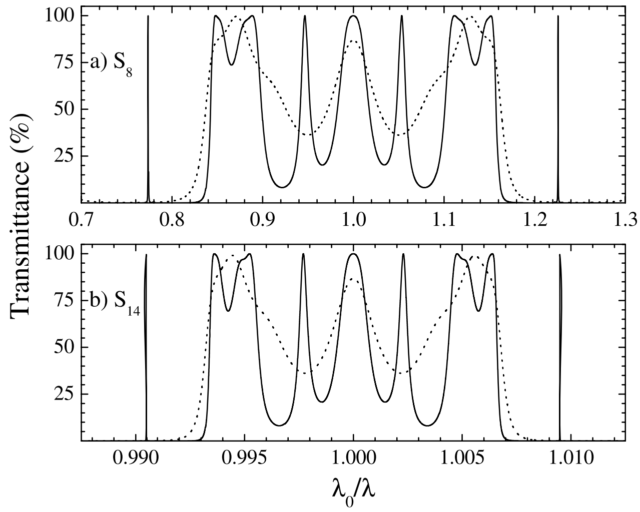

Figure 2. Comparison between the self-similar spectra in single (dotted line) and asymmetric (solid line) Fibonacci multilayers for generations (a) S_8 and (b) S_{14}. Notice the change of scale between graphs (a) and (b).

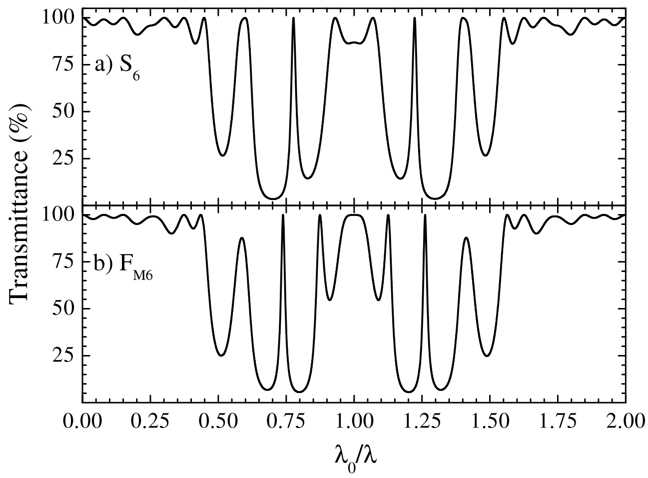

Figure 3. Transmittance spectra of the asymmetric array (a) S_6 and the mirror symmetric array (b) F_{M_6}. Notice that the frequency positions of perfect transmission peaks are different for the two cases.

Let us continue discussing the constant of motion J. When $\delta = m\pi$, i.e. $\lambda_0/\lambda = 2m$, the transfer matrix is equal to the identity matrix (I) for any j and any value u, such that the transmission is perfect, as can be seen directly from equations (6) and (7). In fact, the matrices Q_A and Q_B commute at this light phase and no quasiperiodic effect is present ($J = 0$). However, as is shown in figure 1, in the arrays S_j there are more light phases with perfect light transmission and with quasiperiodic order. We confirmed numerically that in our asymmetric case, perfect transmission peaks take place at light phases where the trace of M is $x = \pm 2$ and the antitrace of M is $y = 0$. One solution that satisfies these matrix properties is the identity matrix I. We propose in our case, as a condition of perfect transmission peaks, that $M_{S_j} = \pm I$. Notice that in general the matrix of F_j is not the inverse of the

Table 1. Hidden symmetry in a truncated Fibonacci multilayer S_j.

S_j	$S_j = \{F_j \mid C_j\}$
S_4	ABAAB | BABBA
S'_4	ABAA | BBAB BA
S_5	ABAABABA | BABBABAB
S'_5	ABAABAB | ABABBAB AB
S_6	ABAABABAABAAB | BABBABABBABBA
S'_6	ABAABABAABAA | BBABBABABBAB BA

Table 2. Hidden symmetry in a permuted Fibonacci multilayer S_j.

S_j	$S_j = \{F_j \mid C_j\}$
S_4	ABAAB | BABBA
S''_4	AABAA | BBABB
S_5	ABAABABA | BABBABAB
S''_5	BABAABAB | ABABBABA
S_6	ABAABABAABAAB | BABBABABBABBA
S''_6	AABAABABAABAA | BBABBABABBABB

matrix of C_j, only the light phases of perfect light transmission satisfy $M_{S_j} = \pm I$. Let us take for example the first non-periodic generation, S_4, where the elements of the transfer matrix M_{S_4} are given by

$$\begin{aligned}
m_{11} = \frac{1}{16u^3}(&\cos^2\delta(3+u+u^2-u^3-4(1+u)\cos 2\delta \\
&+(1+u)^3\cos 4\delta)(-1+u+u^2+3u^3 \\
&-4u^2(1+u)\cos 2\delta+(1+u)^3\cos 4\delta) \\
&-(-1+5u+u^2-u^3+4u(1+u)\cos 2\delta \\
&+(1+u)^3\cos 4\delta)^2\sin^2\delta),
\end{aligned}$$

$$\begin{aligned}
m_{12} = -\frac{1}{8u^3}(&(1+u)\cos\delta(-1-(u-4)u \\
&+4u\cos 2\delta+(1+u)^2\cos 4\delta) \\
&\times(3+u+u^2-u^3-4(1+u)\cos 2\delta \\
&+(1+u)^3\cos 4\delta)\sin\delta),
\end{aligned}$$

$$\begin{aligned}
m_{21} = \frac{1}{16u^3}(&(1+u)(-1-(u-4)u+4u\cos 2\delta \\
&+(1+u)^2\cos 4\delta)(-1+u+u^2+3u^3 \\
&-4u^2(1+u)\cos 2\delta+(1+u)^3\cos 4\delta)\sin 2\delta)
\end{aligned}$$

and

$$\begin{aligned}
m_{22} = \frac{1}{16u^3}(&\cos^2\delta(3+u+u^2-u^3-4(1+u)\cos 2\delta \\
&+(1+u)^3\cos 4\delta)(-1+u+u^2+3u^3 \\
&-4u^2(1+u)\cos 2\delta+(1+u)^3\cos 4\delta) \\
&-(-1+u+5u^2-u^3+4u(1+u)\cos 2\delta \\
&+(1+u)^3\cos 4\delta)^2\sin^2\delta).
\end{aligned}$$

If M_{S_4} is set to be equal to I and the system of equations is solved, the positions of perfect light transmission are given by

$$\begin{aligned}
\delta = 0, \pi, \arccos \pm\sqrt{\Bigg(}&\frac{1}{2(1+2u+u^2)}+\frac{u}{2(1+2u+u^2)} \\
&+\frac{u^2}{2(1+2u+u^2)}-\frac{\sqrt{1+u+u^2+u^3+u^4}}{2(1+2u+u^2)}\Bigg), \\
&\arccos\Bigg[\pm\sqrt{\Bigg(}\frac{1}{2(1+2u+u^2)}+\frac{u}{2(1+2u+u^2)} \\
&+\frac{u^2}{2(1+2u+u^2)}+\frac{\sqrt{1+u+u^2+u^3+u^4}}{2(1+2u+u^2)}\Bigg)\Bigg].
\end{aligned}$$

It has been suggested that perfect transmission peaks in Fibonacci arrays are a result of a mirror symmetry [13, 14]. We have shown that this condition is not necessary. However, we do not discard the possibility that non-evident or hidden symmetries can be involved in our proposed Fibonacci arrays S_j. To analyze hidden symmetries, we consider two possibilities. The first one is illustrated in table 1 for generations 4–6. If we move one position to the left of the joining interface (|) of F_j and C_j, then a kind of mirror symmetry is found if the last two layers on the right side are removed, this is indicated as S'_j. For every A in the left side from the new reference interface (|) one finds a B in the right side and vice versa. One could suppose that as the number of layers increases the effect of taking off the last two layers is negligible. However, we found numerically that the truncated structures give rise to multiple peaks, but not to perfect transmission, that is $T \neq 1$. This means that the complete Fibonacci structure is needed to produce transparent states.

In the second case the last layer of C_j (right side) is moved to the first position of F_j (left side), as illustrated in table 2 for generations 4–6, indicated as S''_j. It is important to mention that if we continue this specific permutation, all the resulting sequences satisfy the condition of joining a sequence with its conjugated one. For instance, the initial S_5 sequence is ABAABABA | BABBABAB, the result of the first step is BABAABAB | ABABBABA, where the first segment is the conjugate of the second segment; the result of the second step is ABABAABA | BABABBAB, which again preserves the conjugated structure. The procedure can be easily continued and one can verify that in all resulting sequences the conjugated structure is preserved. In this case, this hidden symmetry together with the Fibonacci order gives rise to multiple transparent states and their positions are at the same light phase as in our asymmetric Fibonacci array. Although in this asymmetric structure the BB dimers appear, they do not introduce different specific resonant features because the optical path is the same as the AA dimers. One can conclude that if the asymmetric array has a hidden symmetry, although not a mirror one, the Fibonacci arrays produce multiple transparent states.

The invariance of such transparent states under these permutations may be explained in terms of the transfer matrix trace. Since these transparent states are obtained when the transfer matrix is equal to the identity matrix, the trace of such states must be ± 2 because it is an invariant under unitary transformations. Then using the cyclic property of the trace of a product, i.e. $\mathrm{tr}(DEF) = \mathrm{tr}(FDE) = \mathrm{tr}(EFD)$, it is clear that any permuted sequence obtained with this procedure leads to the same transparent states, although the whole spectra are slightly different. We need to emphasize that this mathematical fact is valid for all kinds of sequences. Thus in our proposed array S_j the structure is preserved under this kind of permutation.

The existence of transparent states in the cases of the mirror and asymmetric arrays can be understood in a simple way as a result of joining the transfer matrix of a single Fibonacci multilayer with its inverse for a specific wavelength. For example, if we have a given arbitrary sequence of transfer matrices DEF, its inverse is $(DEF)^{-1} = F^{-1}E^{-1}D^{-1}$. When they are joined one has $(DEFF^{-1}E^{-1}D^{-1})$, which is like having a mirror symmetry. However, in the present case, from equation (1) is clear that the inverse is equal to traversing the interface AB in the opposite direction. Thus, the mirror symmetry allows us to build an effective medium that acts in the wave as the inverse of the single Fibonacci multilayer for certain wavelengths. The conjugated mirror basically works in a similar way, because its matrix structure is analogous to the pure mirror due to the symmetry of the invariant J when A is replaced by B and vice versa in equation (12), and also $T_{ij}^{-1} = T_{ji}$.

4.2. Experimental details

In figure 4 the experimental (solid line) and theoretical (dashed line) reflectance spectrum at an angle of incidence of 5° of the asymmetric Fibonacci multilayer of generation S_6 is shown. A good agreement between experiment and theory is observed as a general trend, except by a shift around $\lambda_0/\lambda = 1$, which increases as the wavelength decreases. This result is a consequence of the refractive index dispersion of PS from the visible to the ultraviolet (UV) region. Deviations from the quarter wavelength in the light phase are expected due to the refractive index dispersion and as we said it limits the study to small Fibonacci generations. Notice that in general, the experimental peaks where perfect transmission is expected to occur are well predicted by the model. Also, for $\lambda < \lambda_0$, a significant reduction in the reflectance intensity is observed due the optical absorption of PS as a function of the wavelength. Experimentally it is difficult to show the auto-similarity of the spectra of the asymmetric Fibonacci arrays because it occurs strictly every six generations, which means that we had to compare, for example, a structure of 16 layers with one of 288 layers. However, we consider that the experimental results show that the proposed asymmetric arrays can give rise to high transmission peaks in real systems in spite of the fact that the ideal perfect transmission cannot be reached.

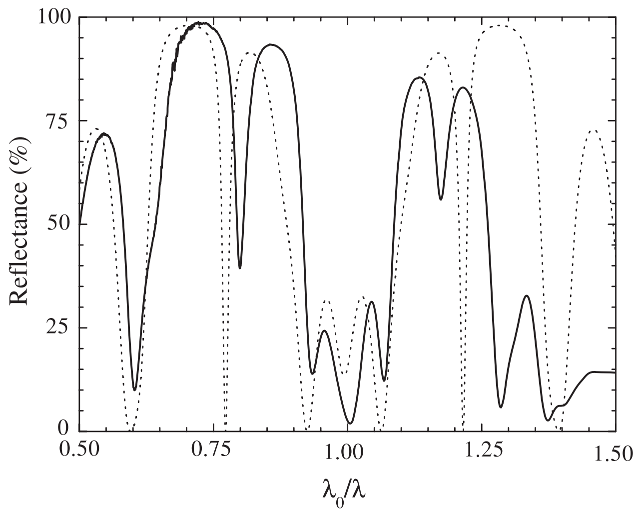

Figure 4. Experimental (solid line) and theoretical (dashed line) reflectance spectra of the asymmetric Fibonacci array S_6 made of porous silicon, where $\lambda_0 = 600$ nm.

5. Conclusions

We proposed and built arrays of Fibonacci dielectric multilayers without mirror symmetry that produced perfect transmission of light at different wavelengths and preserved the rich quasiperiodic properties, such as a self-similar spectra every six generations and the scaling factor. Experimentally, the general trends of light propagation through these quasiperiodic structures, at least for small generations, were verified. The main conclusion of this study is that the mirror symmetry in arrays of dielectric layers in Fibonacci sequences is a sufficient condition but not a necessary one to generate multiple perfect transmission peaks. We have found a condition for perfect transmission in our asymmetric case as $M_{S_j} = \pm I$. Further studies must be performed to see how general this condition is. Our results open broader applications of quasiperiodic systems as filters and microcavities of multiple frequencies.

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